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# “DECIMAL”: A Comprehensive Approach of Teaching Undergraduate Mathematics General Education Courses

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**Abstract:** The teaching and learning of university mathematics general education courses plays a vital role in forming the basic knowledge structure and cultivating the innovation ability of undergraduate students. Innovation and reform of teaching and learning is an eternal topic for improving instructors’ teaching quality and students’ learning experience. Aiming to enhance undergraduate mathematical literacy, we propose a comprehensive teaching approach named “DECIMAL”. The general objectives of this teaching approach are towards a research-oriented teaching model and critical thinking cultivation. Practical experiences are accumulated by years of teaching undergraduate mathematics general education courses, such as “Linear Algebra” and “Probability and Statistics”. Based on current literature research and practical teaching and learning experiences, some guidance of teaching and learning innovation is presented from several aspects, which forms the comprehensive approach “DECIMAL”, consisting of “Design of courses, Examples selection, Charm display, Intelligent tools utilization, Miscellaneous resources, Arousal of interest and Linkage of different courses”. Each factor is illustrated by methodology discussions and specific examples in various teaching scenarios. Positive feedback from students and peers is achieved in teaching practice by applying these teaching methods and techniques comprehensively. The proposed approach has practical significance in teaching and learning of undergraduate mathematics courses for non-math major students.

**Keywords:** University Mathematics, General Education Courses, Mathematical Literacy, Teaching and Learning Innovation, Teaching Design

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## 1. Introduction

University education focuses in general on cultivating the academic skills that enrich personal and professional lives of students. It is critical for mathematics education today to help students in various ways ranging from obtaining necessary mathematical knowledge, forming basic mathematical reasoning skills, solving real-life problems, to improving their mathematical literacy.

There are multiple ways to define mathematical literacy during different learning phases [1, 2]. Educators at university level usually take mathematical literacy as mathematical thinking and application capabilities, such as mathematical modeling from practical problems, computing and analyzing, comprehension and use of modern mathematical language and symbols, and utilizing mathematical software to learn and use mathematics. Therefore, undergraduate mathematics general education

carries out teaching focusing on improving the mathematical literacy of students and helping them enhance intuition, logical thinking ability and even creativity.

The formation of rational thinking pattern, which constitutes special qualities of individuals, is not easily achieved in other courses. However, if mathematics teaching is only regarded as delivering hundreds of theorems and formulae, it may be inevitable to force learners to lose interest and mathematics may become difficult to play its vital role in practice. In fact, students can speak the common language of modern science through rigorous mathematical training and edification of mathematical culture. By mastering the methodology and essence of mathematics, even a few formulae can be used to express vivid conclusions showing endless power.

Undergraduate mathematics general education courses are essential for cultivating mathematical literacy. For this purpose and based on literature research on mathematical literacy [1, 2], students’ learning attitude [3-5] and current

teaching methodologies [6-9], we first present our understanding of the teaching objectives, and then propose a comprehensive way in teaching mathematics courses for non-math major students and illustrate the idea in detail by presenting examples in various teaching scenarios.

## 2. General Objectives of the Teaching Approach

### 2.1. Research-Oriented Teaching Model

Innovative ability is one of the most valuable, important, and highest-level capabilities of mankind. Mathematics is a discipline that combines rigor, logic, precision, creativity and imagination. One of the features of innovative thinking is highly abstract. Using the characteristics of mathematical abstraction, once the abstract thinking ability is formed, the thoughts will have a certain depth so as to achieve the purpose of innovative thinking. Mathematics also cultivates non-logical thinking such as prediction, conjecture, intuition and epiphany, which makes thinking broad, flexible, critical and independent, thus stimulating students' creative thinking ability.

The research-oriented teaching model is a combination of teaching and scientific research. It can cultivate students' innovative spirit and ability through the process of "learning, researching and practicing". As far as the teaching of mathematics general education courses is concerned, there are two ways to explore the research-oriented teaching model. The first is based on the teaching design of each course. Teachers can design in each section some exploratory questions, some of which have exact answers while others are open questions. Through the study of these problems, students can deepen comprehension of the knowledge acquired and cultivate the spirit of exploration and innovation simultaneously. The second way is the training of mathematical modeling ideas. By solving some practical problems that are simplified and abstracted from real life, students can improve their ability to comprehensively apply the knowledge learned to solve practical problems.

### 2.2. Critical Thinking Cultivation

Independent thinking that makes people develop criticism is also vital to education. Albert Einstein once said, "The development of general ability for independent thinking and judgement should always be placed foremost, not the acquisition of special knowledge." [10] Focusing on teaching the key points of knowledge without developing the ability of independent and critical thinking makes it difficult to bring up talents with leadership and innovation.

The introduction of the history of mathematics can be a good way for cultivating critical thinking. There are many examples in mathematics history questioning the traditional thoughts and realizing radical change of ideas. For instance, the birth of Nikolay Ivanovich Lobachevsky's non-Euclidean geometry is the product of questioning the parallel axiom of

Euclidean geometry. More examples such as the establishment of the axiomatic system of probability theory by Andrey Nikolaevich Kolmogorov and the evolution of central limit theorem are also interesting and inspiring stories.

The implementation of mathematics teaching itself can also be developed to foster critical thinking. Teachers may occasionally embed some errors intentionally in the lectures for students to discover. Passive listener and recipients can become active skeptics and explorers. To create the atmosphere that encourages students to challenge authority, textbooks or instructors, the design of teaching and learning is particularly important.

To this end, it is necessary to innovate teaching methods and means while inheriting good ideas from traditional teaching.

There have been various teaching methods both in education theory and in teaching practice. For example, heuristic teaching [8] and inquiry teaching [9] are typical teaching modes for cultivating innovative and critical thinking. Based on the teaching purpose, contents, students' level and knowledge structure, heuristic teaching may help students study actively and promote mental development by using inspiration-inducing methods to deliver knowledge and train ability. Inquiry-based teaching means that when students learn to master new concepts and principles, teachers only offer them some examples and problems, allowing students to explore independently through reading, listening, observation, experiment, discussion and thinking to discover the corresponding conclusions. Teaching pedagogies have been evolving all the time and they can bring new options and opportunities for teachers and students.

## 3. DECIMAL: A Comprehensive Approach of Teaching Mathematics

On the foundation of existing teaching methodologies, some experience is accumulated through trials of teaching practice. We summarize it as a comprehensive approach for teaching and learning named "DECIMAL", which comes from the initials of seven specific teaching and learning suggestions.

### 3.1. Design of Courses

The implementation of active psychological and intellectual guidance for students is a high-level behavior that enlightens wisdom, develops savvy, and discovers potential. Mathematical teaching should reflect the creative process of mathematics. This requires an optimized combination of the contents of different textbooks in a way that is convenient for students to associate and discover "new knowledge". We have probed into multiple international textbooks for undergraduate mathematics general education, which show distinct styles of illustration. In order to meet the requirements for critical thinking cultivation and reach a moderate degree of theoretical depth, teachers can combine

the advantages of different types of textbooks to design appropriate curricula. For example, we used several international textbooks for teaching linear algebra course at freshmen level [11-15], trying to ensure the balance between the depth of linear algebra theory and the breadth of applications. Taking advantage of various textbooks makes it possible to help students lay a solid mathematical foundation and obtain the corresponding mathematical literacy. For students whose native language is not English, it is also a good opportunity for them to adapt to the bilingual environment.

Course design must combine the characteristics of both the subject and the students. Taking "Linear Algebra" as an example, the course is basic, abstract, systematic and practical. The students are usually freshmen with good mathematical foundation from high school and strong desires for knowledge. We design the student-oriented lectures including intensive theory with applications and aim to teach students in accordance with their aptitude. To be specific, we adopt the following principles: a) the class is problem-driven and focuses on guidance; b) let students ask questions and create an atmosphere of participation; c) teachers introduce mathematics history occasionally and help establish cognitive concepts; d) encourage and communicate with students and cultivate harmonic teacher-student relationship; e) use diversified instructional means to avoid boring class.

We take the teaching of "eigenvalues and eigenvectors: concept and calculation" as an example. The teaching design is fulfilled in three levels. In cognitive aspect, students should understand the concept of eigenvalues and eigenvectors, and master the method for computation. In terms of performance, the ability of analyzing and solving real-life problems should be trained. Last but not least, in the emotional aspect, we should try to cultivate the spirit of exploration and innovation.

Based on the ideas above, we start this lecture from an introductory example: "migration between city and suburbs" [11]. Analyzing this problem leads to the definitions of eigenvalues and eigenvectors, so that students feel like they discover the concept by themselves. More guidance is offered to find the method for calculation by using existing knowledge. Finally, the beauty of Fibonacci series and golden ratio are presented in front of the students, who are further inspired to explore the fascinating evolution trajectories of difference equations by applying the concepts of eigenvalues and eigenvectors they have just learned. Such kind of teaching design makes the lecture content richer and more acceptable.

### 3.2. Examples Selection

Picking up appropriate examples is an important issue to take into consideration when preparing lectures. Many examples in mathematics general education courses contain the idea of mathematical modeling. "Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena." [7] The training of mathematical modeling

brings benefits to students' comprehensions of the real world, and helps non-math major students perform better in future engineering projects.

Due to the compact arrangement of syllabus and the limitation of students' knowledge system structure, it is almost impossible to teach specialized modeling methods during class time. However, the idea of modeling can be embedded in case studies. This depends on the selection of examples and good guidance.

For example, in the course "Probability and Mathematical Statistics", we discuss the following question revised from an example in the textbook by John A. Rice [16]. Suppose there is a batch of human serum samples. Assume that the probability of containing hepatitis virus in each sample is 0.3%. If we mix the serum samples of 100 people, try to calculate the probability of containing hepatitis virus in the mixed sample. The original intention of this example is to explain to students the concept of independent events and the properties in probability calculation. If we stop at the end of solving the problem, students may understand the technique of calculation only. Now we can move further to ask questions: Why should we mix 100 people's serum? What is the purpose? Is it just for fun or just for exercise? This will lead to deeper thoughts. In order to give the test result for each person's serum, is it a good way to mix all 100 people's serum? Is it wise to test each individual's serum sample separately? If mixing all means high risk of getting a positive result, and testing separately yields low efficiency, what is the best way out? Can we divide the 100 samples into two groups? What will happen if we divide them into three groups? By taking both the probability of containing virus in the grouped sample and the costs of testing into account, how can we design the optimal grouping method to reach the tradeoff? Students are encouraged to establish a mathematical model and even perform programming to validate the model after class. These exploratory questions can trigger students' thinking and cultivate their modeling ideas besides deepening their understanding of basic concepts. In such a way, the classroom teaching can get positive effects by using only a little more time than the traditional mode.

### 3.3. Charm Display

Research shows that "math anxiety" is a common phenomenon among all levels of mathematics education throughout the world [4, 5]. Some students fear to learn mathematics. Students often ask us the use of learning a certain course. The usual explanations include answers such as "mathematics is a kind of brain gymnastics", "we should learn more mathematics to improve our literacy", "you will gradually feel it with the accumulation of knowledge". Actually as teachers we should also question ourselves if we can let students see some practical applications just inside the course. Showing the charm or power of mathematics is a useful way to overcome anxiety.

For example, the singular value decomposition (SVD), as part of the linear algebra course, seems esoteric to freshmen. However, it is a very powerful tool that can be applied in

many aspects such as image processing and data analysis [13].

After explaining the basic principles and calculation procedure of SVD, we can make an extension and give students a taste of the application in image processing. Suppose we have an image with the resolution  $m \times n$ , then we need a  $m \times n$  matrix to save it. With the singular value decomposition, only the singular values of the matrix and corresponding eigenvectors need to be stored, and the storage taken is  $r \times (m + n + 1)$ . With the number of singular values  $r$  much smaller than  $m$  and  $n$ , the storage amount required to store the image is reduced dramatically. Furthermore, if the singular values of the matrix are ordered decreasingly and show a remarkable drop after a certain number, then some of the small singular values can be removed so that the image is further compressed, and the image noise reduction effect is achieved without much distortion.

There are many details about the usage of SVD, but through such kind of examples, students may feel that they can use some of the knowledge they learn during the semester to solve some research problems and hot issues. If we show them the charm and power of mathematics in a timely manner, the efforts become worthwhile. Some open-ended questions that seem to be far-reaching and interesting are also something solvable at fingertips. After students have a sense of accomplishment, they will be encouraged and willing to invest more time on conducting in-depth research.

### 3.4. Intelligent Tools Utilization

Technology enhanced learning or “e-learning” is a very hot topic in last decades [17-20]. The purpose of using technologies in classroom is to improve student engagement and to have a better control and leading of learning behaviors [21].

Classroom voting feedback systems such as “iClicker” and “Sunvote” can get students to class and engage them with polling and quizzing. These simple tools can enhance students focus and identify their misconceptions instantly. In addition to improving student attendance, teachers can also get to know to what extent each student has mastered the concepts, so as to adjust the progress and make better use of the time in class. For instance, when explaining how to calculate the determination of block matrices, we use the voting system to raise a question that students easily make mistakes about. Getting feedback in real time can give the instructors some guidance to lead the class. This enables a better interactive atmosphere in the classroom and the students will have deeper impression on the specific question.

With the development of information technology, intelligent teaching platforms have been emerging rapidly. Typical and influential tools include the “uReply” system developed by the Chinese University of Hong Kong and promoted in Hong Kong universities, “Rain Classroom” plugin developed by Tsinghua University and used globally

in over 10,000 universities and institutions. These tools can change the teaching and learning dynamics through facilitating various types of interactions in the classroom. Starting fall semester 2017, we have been using the smart classroom tool “Rain Classroom”, which penetrates all phases of teaching and learning. By tracking “before class – in class – after class” study of the class and the behavior of each student, we can better understand students’ needs, and even give warnings to slow learners through the learning progress. This has changed the existing situation of determining the grades by final exams only and can offer better help to students on demand. These student-oriented tools are widely welcomed by students and teachers and have significantly improved the teaching effect.

### 3.5. Miscellaneous Resources

Teaching is a kind of comprehensive art. Teachers should learn from different disciplines and take advantage of all possible resources to make teaching more active, effective and attractive. Here are some possible ways.

#### 3.5.1. Combining the Research Work

University teachers usually devote to both teaching and research. Research, although taking up a lot of time and efforts of teachers, should not be treated as a rival of teaching. As a matter of fact, teaching and research can be mutually improved. By combining with the teacher’s own research work, students are exposed to some of the research issues at the forefront. Take linear algebra as an example, matrix manipulation is widely used in scientific problems such as super-computing, webpage searching engine, machine learning, solution of large-scale linear equations in satellite precise orbit determination, and so on. For students who have spare time and energy, appropriate guidance can lead them to research opportunities.

#### 3.5.2. Communicating with Students via Multiple Channels

Nowadays, connecting with students is no longer fixed to office hours as many years ago. There are many platforms (“Blackboard”, “Sakai”, etc.) to publish courseware, check assignments, and answer questions online. Some instant chatting tools (such as QQ, Wechat, Skype, etc.) can also provide students with quick response Q&A. The SPOC (Small Private Online Course) platform implements a combination of online and offline teaching modes. In addition to sharing resources with students, instructors can also observe the progress and activities of students, and encourage students to learn from peers.

#### 3.5.3. Making Full Use of Open Courseware

“Internet Plus Education” policy provides a way to relieve the education imbalance and inequality. Living in an era of advanced technology, teachers can take advantage of miscellaneous online resources.

Quality online resources, ranging from the earliest MIT Open Courseware, to the development of edX, Udacity, Coursera, provide a chance for life-long learning. China also have various MOOC (massive open online courses) resources,

such as XuetangX, iCourse, Chaoxing Erya, Netease Open Courses. These platforms have online materials, including video courseware, syllabus, and offer online exercises, exams and feedback. The characteristics and forms are increasingly diverse with the aim of achieving high-quality resources sharing. In the process of globalization of university mathematics general education courses, traditional curricula have been influenced in a positive way by advanced education concepts from all over the world. Teachers should try to gain experience from contemporary international curriculum resources, and apply excellent educational concepts and pedagogies into daily teaching.

### 3.6. Arousal of Interest

Albert Einstein once said, "It is the supreme art of the teacher to awaken joy in creative expression and knowledge." [10] Interest is the driving force of innovation. Therefore, one of the main tasks of university mathematics teaching is to cultivate students' positive and innovative interests. Teaching needs to enable students to proactively discover knowledge independently and apply knowledge creatively. Otherwise, without initiative forces, mathematics instructors just try desperately to "sell a product to a market that doesn't want it, but is forced by law to buy it." [22]

The impression of mathematics to many people is rigid and boring. Some students have not changed their minds and only felt more difficult and uninterested after studying mathematics for years. This requires finding the root cause from teaching. Interest is the best teacher and the most original motivation for students to learn new things and pursue innovation. It is crucial to inspire their intuition and insight. Using the intuitive image of mathematics in teaching is an important approach to stimulate students' curiosity. Making good use of multimedia with excellent class design can also attract students' attention and make the classroom vivid.

For example, in the application of the central limit theorem in probability theory, after explanation of theoretical knowledge, instructors may combine examples in life, such as the Galton board, which is intuitive and easily understood. Why do the balls form a shape of the normal distribution density function? What is the theoretical basis? And can we determine the parameters of the distribution? Why do we assume random indices to follow normal distribution in most science and engineering problems? This example can help to persuade students that mathematics can explain natural phenomena in their lives.

### 3.7. Linkage of Different Courses

Although each branch of mathematics has its own characteristics and specific research methods, mathematical knowledge is not isolated. If teachers can integrate and make connections of different subjects on certain issues, they will consolidate students' understanding of knowledge. We illustrate by several examples as follows.

#### 3.7.1. Example 1: Quadratic Forms

Linear algebra can provide powerful tools to study the

features of quadratic multivariate functions, which also take a main role in calculus. Learning the theory of quadratic forms is an essential component for deeper understanding of the theorems in linear algebra. It also helps students to get a visual sense of quadratic surfaces in analytic geometry and continuous functions in calculus.

For example, let  $f(x_1, x_2, \dots, x_n) = x^T A x$  be a real quadratic form, where  $x$  is a  $n$ -dimensional real vector and  $A$  is a  $n \times n$  real symmetric matrix. Suppose we know that there are  $n$ -dimensional real vectors  $x_1, x_2$  such that  $x_1^T A x_1 > 0$  and  $x_2^T A x_2 < 0$ . Prove that there must exist a  $n$ -dimensional real vector  $x_0 \neq 0$ , such that  $x_0^T A x_0 = 0$ . [15]

This problem can be explored from the perspective of both calculus and linear algebra. To advocate divergent thinking from various angles for a given problem is a beneficial practice. It can help students understand the comprehensive knowledge points and find links between different mathematics courses.

#### 3.7.2. Example 2: Invertible Matrices

Question: If you put 1s and 0s at random into the entries of a  $n$  by  $n$  matrix, is it more likely to be invertible or singular? [13]

This problem seems simple, but it becomes hard as the matrix order increases. In addition to theoretical analysis, statistical simulations can be employed to analyze the invertibility of large-scale random binary (0-1) matrices and calculate the proportion of the invertible matrices. To estimate the invertibility of the random matrices, it requires simulation strategies design based on different sampling methods. This kind of discussion uses the knowledge of linear algebra, the method of probability theory and mathematical statistics, and the experience from programming. It is of great benefit to improve students' comprehension of the essentials of the problem and enhance their ability to solve it. The difficulty is moderate for undergraduate students and the problem encourages students to think about matrix properties, programming techniques, and the relationship between theoretical analysis and actual calculations.

#### 3.7.3. Mathematics and Other Subjects

Mathematics lays a foundation for science and engineering subjects. There is vast amount of problems connecting mathematics and physics, and can be good examples for teaching difference equations and ordinary differential equations. Convolution operation in statistical signal processing is based on the distribution of random variables in probability theory. Concepts of conditional expectation and conditional variance are often applied in finance. Inter-disciplinary problems are good materials for each subject and making use of them can be a way to enhance mathematics teaching.

## 4. Conclusion and Recommendations

Teaching and learning of undergraduate mathematics general education courses is an eternal topic and plays an irreplaceable role in the cultivation of undergraduates. In

order to stimulate students' enthusiasm and potential for innovation, teachers need to innovate mathematics teaching. Pedagogies are diverse and teachers have their own styles. On the basis of mastering the basic teaching rules, innovation can be realized based on the instructor's own personality and the subject characteristics. We discussed a comprehensive approach, "DECIMAL", from the practical teaching experience. Teaching of all levels should be student-centered. The final goal is to help students master the skills in the specific subjects and improve their professional literacy. Teachers need to review the feedback from students on a regular basis and update teaching methods, since the involvement of education and the enhancement of learning and teaching never end.

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